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A Normal-Weighted Exponential Stochastic Frontier Model

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Abstract

This thesis introduces a new stochastic frontier model called a normal-weighted exponential stochastic frontier model. We have derived a closed form log-likelihood function and JLMS inefficiency estimator of a normal-weighted exponential stochastic frontier model. In addition, we have derived the gradient and hessian matrix of a normal-weighted exponential stochastic frontier model. A Monte Carlo (MC) simulation is carried out to verify the correctness of the derivations, of a normal-weighted exponential stochastic frontier model, and to study the finite sample properties of maximum likelihood estimator. Our simulation result shows that a normal-weighted exponential stochastic frontier model performs well compared to a normal-exponential stochastic frontier model. In our simulation result, it shows that as sample size increases, the bias and standard errors decrease. Furthermore, a real-world data application is carried out, with the goal of estimating the carbon efficiency of African manufacturing firms. We have estimated an input-requirement production function, using fuel consumption as a dependent variable and output and other inputs as independent variables. Our estimated result shows that the estimates of coefficients are the same across models. However, there are differences in the carbon efficiency estimates of manufacturing firms. We have used the carbon efficiency estimates to rank African countries, and Egypt is the most carbon efficient country in Africa. We have also run multiple linear regressions on carbon inefficiency estimates to see the determinants. In all three stochastic frontier models, top manager work experience, obstacles to accessing finance, firm size, export status, and foreign ownership are the key determinants.

Keywords: Inefficiency estimator, stochastic frontier model, weighted exponential distribution

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Introduction

The Stochastic Frontier Analysis (SFA) is a widely used methodology for the measurement of productivity and efficiency of decision making (DM) units, either firms or countries. Using stochastic frontier models, it is possible to estimate the production efficiency, cost efficiency, or profit efficiency of a firm. Kumbhakar and Lovell (2000) have a book-length discussion about the applicability of a stochastic frontier analysis to efficiency measurement of a firm, using the production function, cost function, and profit function. A firm is fully efficient if it can produce the maximum possible output for a given technology and cost, or if it can attain the minimum possible cost for a given level of output and technology. Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA) are widely used approaches for the efficiency analysis of firms. These approaches, with many extensions, are widely discussed in Coelli, et al. (2005). The general framework of stochastic frontier models for estimating the efficiency of firms was introduced by Aigner, et al (1977) and Meeusen and Broeck (1977).

An exponential distribution and a half-normal distribution are commonly used assumptions for the inefficiency part in stochastic frontier analysis. The advantage of using a half-normal distribution and an exponential distribution is that they provide a closed form likelihood function for a stochastic frontier model. However, there are also limitations of using the exponential distribution and the half-normal distribution; see e.g., Greene (2003) and Stevenson (1980). The first limitation is that both the half-normal distribution and exponential distribution have a mode value of zero. Which means the model assumes a highest probability value for zero value of inefficiency score. Alternative statement is that firms are assumed to be fully efficient a priori. Another limitation is that both probability distributions are governed by a single parameter, a scale parameter. Moreover, both distributions are asymmetrically positively skewed and assumes very low probability of being inefficient Carree (2002). Due to these limitations various probability distributions are assumed for the inefficiency score.

Hajargasht (2015) introduce a Rayleigh distribution into a stochastic frontier model. A Rayleigh distribution is a one-parameter distribution, and it has a non-zero mode. Papadopoulos (2021) present a single parameter generalized exponential distribution and the mode of the distribution is away from zero. These non-zero mode distributions can represent cases where the highest probability is non-zero in efficiency scores. However, since they are governed by a single parameter, they are less flexible. In fact, any probability density function defined on positive value can be a candidate model for the inefficiency score.

In order make the inefficiency score more flexible various two parameter distribution are introduced. The gamma distribution is proposed in (Stevenson (1980), Beckers and Hammond (1987) and Greene (2003)). The Weibull distribution and the beta distribution is introduced in Tsionas (2007) and Tsionas (2012), respectively. However, assuming these flexible distributions for the inefficiency part creates a problem of deriving a closed form likelihood function.

In this thesis a new probability distribution, a weighted exponential distribution, is introduced for the inefficiency part of a stochastic frontier model. The weighted exponential distribution is a flexible distribution with two parameters, shape parameter and scale parameter. The weighted exponential distribution is flexible as the gamma distribution. However, the advantage of the weighted exponential distribution is that it enables to get a closed-form likelihood function and inefficiency estimator. The weighted exponential distribution contains the one parameter generalized exponential distribution as its special case. Another argument for introducing a new probability distribution is based on the aphorism in statistics saying that all models are wrong, but some are useful. Therefore, it is recommended for the practitioners to have the weighted exponential distribution in their tool kit and verify its usefulness based on the data (Gupta & Kundu, 2009).

Stochastic Frontier Analysis

The stochastic frontier model is a popular methodology for measuring inefficiency of a firm. A firm is efficient if it produces the maximum possible output for a given cost or achieves the minimum possible cost for a given level output. The stochastic frontier model is the measurement of production function or cost function with symmetric statistical error and inefficiency score. Both Aigner et al. (1977) and Meeusen and van den Broeck (1977) assumed the statistical error to follow the normal distribution with zero mean and constant variance. For the inefficiency score different probability distribution have been proposed.

Thus, the specification of a stochastic frontier model with production function and two multiplicative error terms of the efficiency level U and the statistical error V is

$$Y\mathrm{i} = f\ (\boldsymbol{xi};\theta)\ UV$$

where $U \in [0, 1]$, $U \in [0, \infty]$, $f\ (\boldsymbol{xi};\ \theta)$ is the production function. Any deviation from the production function is assumed to be the result of two disturbances, the statistical error V and efficiency level U. The efficiency level U is the ratio of actual output to potential output, that is

U = yi

Thus, by logarithmic transformation, the stochastic frontier model in additive form is

$$ln\ Yi = \ ln(f(\boldsymbol{xi};\ \theta)) + \ v - \ u$$

where $v = ln\ (V)$ and $u = -\ ln(U)$.

Once the functional form of the production function is stated, the next step is to propose distributional assumptions for the inefficiency score u and the statistical error term v. The combined error term ε, in case of production frontier, is defined as

$$\varepsilon = v - u,$$

where u is the inefficiency score and v is the statistical error. Having a tractable probability density function of ε depends on the choices of probability distribution made for u and v, and on the assumption whether u and v are independent or not. Assuming independence between the statistical error v and inefficiency score u, the joint distribution of v and u is given by

$$f(u, v) = \ f(u)f(v),$$

where $f(u)$ and $f(v)$ are the probability density function of u and v, respectively. The next step is to substitute one of the random variables from the combined error $\varepsilon = v - u$, and integrate the remaining variable. We have

$$f(\varepsilon) = \int_{-\infty}^{\infty} f(v - \varepsilon) f(v) dv,$$

For some choices of parametric distributions u and v, the integral above can be computed and we have a closed-form distribution function of ε, Both the pioneering articles of Aigner, et al. (1977) and Meeusen and Broeck (1977) and most subsequent applied researchers have assumed the random error v to follow a normal (Gaussian) distribution with zero mean $\mu = 0$ and constant variance σ2. For u, different probability distributions have been proposed. Only a few of the results in a closed-form probability density function of ε.

Once the probability density function of ε is derived, then the likelihood function of the stochastic frontier model for N observation is

$$L(\theta \mid \varepsilon) = \prod_{i=1}^{N} f(\varepsilon \mid \theta),$$

where $\varepsilon= v- u= \log(y) - \log [f\ (x; b)]$.

The vector of parameters θ contains the coeffects of the production function b and parameters from the probability distribution of inefficiency score u and the statistical error v. The production function $f\ (x; b)$ can be either a Cobb-Douglass production function or another production function that is linear after logarithmic transformation. Maximizing the likelihood function requires the likelihood function to be in a closed form, and this has been a challenge when a flexible distribution is assumed for the inefficiency score u.

JLMS Inefficiency Estimator

The primary goal of the stochastic frontier analysis is to have estimates for inefficiency score u. Aigner, et al. (1977) used the mean of u and the maximum likelihood estimators to get the inefficiency estimates of each firm. It is possible to estimate the average inefficiency score $E(\hat{u})$ based on the estimates of the average of composite error $E[\hat{\varepsilon}]$, since $E[u] = E[\varepsilon]$. However, it is also desirable to have inefficiency estimates for each firm and a complete probability distribution for û. As a solution Jondrow, et al. (1982) proposed a method of deriving the conditional distribution of u from the probability density function of ε. The conditional probability density of function of u, $f\ (u|\ \varepsilon)$, is derived from the ratio of the joint probability density of u and ε, $f\ (\varepsilon, u)$, to the marginal probability density of ε, $f(\varepsilon)$.

$$f(u \mid \varepsilon) = \frac{f(\varepsilon, u)}{f(\varepsilon)}$$

Applications of the Stochastic Frontier Models to Carbon Efficiency Analysis

Energy efficiency of a firm or a country is estimated using various methodologies. Carbon efficiency can be analyzed using either the Data Envelopment Analysis (DEA) method or the Stochastic Frontier (SFA). The Data Envelopment Analysis (DEA) can be used to evaluate firm's carbon efficiency from production perspective. Furthermore, they have estimated the effect of financial performance on carbon efficiency. The four major evaluation methodologies of energy efficiency are, the stochastic frontier analysis, data envelopment analysis, exergy analysis and benchmarking comparison (Li & Tao, 2017). Li and Tao (2017) have revied all 4 methods and summarized that the SFA approach as solid fundamental work in modeling application. There are three widely used stochastic frontier methodologies for estimating energy efficiency. These are the input demand frontier functions (Llorca, Banos, Somoza, & Arbue´s, 2017), Shephard input distance function (Hu & Honma, 2014) and input requirement functions. The Stochastic Frontier Analysis (SFA) can be either input oriented SFA or output oriented SFA. The input oriented SFA measures how much extra input is employed to produce a given level of output. While the output oriented SFA measures how much the output falls below the frontier (Jin & Kim, 2019)

Hu and Honma (2019) used the stochastic frontier analysis to estimate the energy efficiency of industries in 14 developed countries. They have used a panel data for the period of 1995-2005 and 10 industries are included. The countries included in the study are United States, United Kingdom, Sweden, Finland, Germany, Italy, the Netherlands, Portugal, Australia, Austria, Denmark, the Czech Republic, Japan, and South Korea. The industrial sectors included in the study are, the construction industry, the food and tobacco industry, the chemical and petrochemical industry, the iron and steel industry, the machinery industry, the paper industry, the non-metallic minerals industry, the textile and leather industry, the wood industry, the pulp and printing industry, and the transport equipment industry. They used 4 variable inputs (labor, capital, energy, intermediate input) and the variable output is measured using the value added. They used the stochastic frontier distance function in which the production function part is specified as Cobb-Douglass production function. Their regression result shows a decreasing efficiency for the industries, construction, paper, and textile. On the contrary, the industry sector shows an increase in efficiency.

The efficiency estimate of more than half industries shows insignificant change. The most efficient performance of industries can be classified into countries. The food industry, the textile industry, and the machinery industry are more efficient in Portugal. The construction industry and the wood industry are better in United Kingdom. While the chemical industries and the paper industries are efficient in Denmark. The rest of industries are distributed as the non-metallic mining in Czech Republic, the transport industry in Italy, and the iron and metal industry in South Korea. Moreover, their result also shows the food industry is the most efficiency with efficiency level of 82.5 percent and the lowest efficiency level is for the wood industry which is 12.7 percent.

A Normal-Weighted Exponential Stochastic Frontier Model

We assume that v follows a normal distribution and u follows a weighted exponential distribution, therefore, we will call this the "normal-weighted exponential stochastic frontier model". The probability density function (pdf) of u is given by

$$f(u) = \frac{(\alpha + 1)}{\alpha} \lambda e^{-\lambda u}\left(1 - e^{-\lambda \alpha u}\right),$$

where α> 0 is the shape parameter, and λ> 0 is the scale parameter.

The statistical error, v, is assumed to be normal with mean 0 and constant variance σ2, which is

$$f(v) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(v)^2}{2\sigma^2}},$$

where σ2> 0.

Assuming independence between the two random variables and setting v= ε− u, the joint density function is given by

$$f(u, \varepsilon - u) = \frac{(\alpha + 1)}{\alpha} \lambda e^{-\lambda u}(1 - e^{-\lambda \alpha u}) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\varepsilon - u)^2}{2\sigma^2}}$$

Integrating the above joint density function with respect to inefficiency score u gives us the marginal density function of composite error ε. Which is

$$f(\varepsilon) = \int_0^\infty f(u, \varepsilon - u) du,$$

$$f(\varepsilon) = \frac{\lambda(\alpha+1)}{\alpha\sqrt{2\pi\sigma^2}} \int_0^\infty e^{-\lambda u}(1 - e^{-\lambda\alpha u}) e^{-\frac{(\varepsilon-u)^2}{2\sigma^2}} du$$

$$f(\varepsilon) = \frac{\lambda(\alpha+1)e^{-\frac{\varepsilon^2}{2\sigma^2}}}{\alpha\sqrt{2\pi\sigma^2}} \int_0^\infty (1 - e^{-\lambda\alpha u}) e^{-\left(\frac{u^2-2u(\varepsilon-\sigma^2\lambda)}{2\sigma^2}\right)} du$$

$$f(\varepsilon) = \frac{\lambda(\alpha+1)e^{-\frac{\varepsilon^2}{2\sigma^2}+\frac{(\varepsilon-\sigma^2\lambda)^2}{2\sigma^2}}}{\alpha\sqrt{2\pi\sigma^2}} \int_0^\infty (1 - e^{-\lambda\alpha u}) e^{-\left(\frac{u^2-2u(\varepsilon-\sigma^2\lambda)}{2\sigma^2}\right)-\frac{(\varepsilon-\sigma^2\lambda)^2}{2\sigma^2}} du$$

$$f(\varepsilon) = \frac{\lambda(\alpha+1)e^{-\varepsilon\lambda+.5\sigma^2\lambda^2}}{\alpha\sqrt{2\pi\sigma^2}} \int_0^\infty (1 - e^{-\lambda\alpha u}) e^{-\frac{\left(u-(\varepsilon-\sigma^2\lambda)\right)^2}{2\sigma^2}} du$$

$$f(\varepsilon) = \frac{\lambda(\alpha+1)e^{-\varepsilon\lambda+.5\sigma^2\lambda^2}}{\alpha\sqrt{2\pi\sigma^2}} \left[\int_0^\infty e^{-\frac{\left(u-(\varepsilon-\sigma^2\lambda)\right)^2}{2\sigma^2}} du - \int_0^\infty e^{-\left(\frac{u^2-2u\left(\varepsilon-\sigma^2\lambda(\alpha+1)\right)+(\varepsilon-\sigma^2\lambda)^2}{2\sigma^2}\right)} du\right]$$

$$f(\varepsilon) = \frac{\alpha+1}{\alpha}\lambda\Phi\left(\frac{(\varepsilon-\sigma^2\lambda)}{\sigma}\right)e^{-\varepsilon\lambda+.5\sigma^2\lambda^2} - \frac{\alpha+1}{\alpha}\lambda\Phi\left(\frac{(\varepsilon-\sigma^2\lambda(\alpha+1)}{\sigma}\right)e^{-\varepsilon\lambda(\alpha+1)+.5\sigma^2\lambda^2(\alpha+1)^2}.$$

For a stochastic frontier production function, we have

$$f(\varepsilon) = \frac{\alpha+1}{\alpha}\lambda\Phi\left(-\frac{(\varepsilon+\sigma^2\lambda)}{\sigma}\right)e^{\varepsilon\lambda+.5\sigma^2\lambda^2} - \frac{\alpha+1}{\alpha}\lambda\Phi\left(-\frac{(\varepsilon+\sigma^2\lambda(\alpha+1)}{\sigma}\right)e^{\varepsilon\lambda(\alpha+1)+.5\sigma^2\lambda^2(\alpha+1)^2}.$$

By rearranging the marginal density function of composite error ε, is given by

$$f(\varepsilon) = \frac{\alpha+1}{\alpha}\lambda e^{\varepsilon\lambda + .5\sigma^2\lambda^2}\left\{\Phi\left(-\frac{(\varepsilon+\sigma^2\lambda)}{\sigma}\right) - \Phi\left(-\frac{(\varepsilon+\sigma^2\lambda(\alpha+1))}{\sigma}\right)e^{\varepsilon\lambda\alpha + .5\sigma^2\lambda^2(\alpha^2+2\alpha)}\right\}.$$

The log-likelihood function for N observation is

$$\begin{aligned}\log L(\alpha,\lambda,\sigma \mid \varepsilon_i) \\ &= \sum_{i=1}^{N}\varepsilon_i\lambda + \frac{N\sigma^2\lambda^2}{2} + N\{\log[\lambda]\} + N\{\log[\alpha+1]\} - N\{\log[\alpha]\} \\ &+ \log\left\{\sum_{i}^{N}\left[\Phi\left(-\frac{(\varepsilon+\sigma^2\lambda)}{\sigma}\right)\right.\right. \\ &\left.\left. - \Phi\left(-\frac{(\varepsilon+\sigma^2\lambda(\alpha+1))}{\sigma}\right)e^{\varepsilon\lambda\alpha + .5\sigma^2\lambda^2(\alpha^2+2\alpha)}\right]\right\}.\end{aligned}$$

JLMS Inefficiency Estimator

The primary goal of the stochastic frontier analysis is to have estimates of inefficiency for each firm. Jondrow, et al. (1982) proposed the conditional distribution of u and the maximum simulated likelihood estimator. The conditional distribution of is derived by dividing the joint distribution of the inefficiency score and the composite error, (ε, u), and dividing by the marginal distribution of the composite error ε.

The conditional probability distribution of u is given by

$$f(u \mid \varepsilon) = \frac{f(u, \varepsilon - u)}{f(\varepsilon)}$$

We can see that the distribution of the inefficiency score is a weighted sum of a truncated normal distribution. That is

$$f(u \mid \varepsilon) = W_1 \frac{e^{-\frac{(u-\mu)^2}{2\sigma^2}}}{\Phi\left(\frac{\mu}{\sigma}\right)\sqrt{2\pi\sigma^2}} - W_2 \frac{e^{-\frac{(u-\mu_*)^2}{2\sigma^2}}}{\Phi\left(\frac{\mu_*}{\sigma}\right)\sqrt{2\pi\sigma^2}}$$

where $W_1 = \frac{\Phi\left(\frac{\mu}{\sigma}\right)}{C}$ and $W_2 = \frac{\Phi\left(\frac{\mu_*}{\sigma}\right)e^{-\varepsilon\lambda\alpha+.5\sigma^2\lambda^2(\alpha^2+2\alpha)}}{C}$ are weights.

We can use the mean of the above inefficacy distribution with maximum likelihood estimator of the parameters, $E\ [\ u \mid \hat{\varepsilon}]$, as an estimator to each firm, which is commonly called the JLMS inefficacy estimator (Jondrow, Lovell, Materov, & Schmidt, 1982). And the mean of the sum of a random variable is the sum of the mean of each random variable.

$$E[\hat{u} \mid \hat{\varepsilon}] = \widehat{W}_1\left(\hat{\mu} + \hat{\sigma}\frac{\phi\left(\frac{\hat{\mu}}{\hat{\sigma}}\right)}{\Phi\left(\frac{\hat{\mu}}{\hat{\sigma}}\right)}\right) - \widehat{W}_2\left(\hat{\mu}_* + \hat{\sigma}\frac{\phi\left(\frac{\hat{\mu}_*}{\hat{\sigma}}\right)}{\Phi\left(\frac{\hat{\mu}_*}{\hat{\sigma}}\right)}\right)$$

Gradients and Hessian Matrix

The maximum likelihood estimators are derived by differentiating the loglikelihood function with respect to parameters. Most programing languages including MATLAB have built-in optimization functions. However, it is also desirable to have a gradients and hessian matrix of likelihood function and use it for numerical optimizations. The log-likelihood function of a normal-weighted exponential stochastic frontier model is given by

$$\log L(\alpha, \lambda, \sigma \mid \varepsilon) = \log(\alpha + 1) - \log(\alpha) + \log(\lambda) + \lambda\varepsilon + \frac{\sigma^2\lambda^2}{2} + \log\{\mathrm{K}\},$$

where $\varepsilon = y - X\beta$, $\mathrm{K} = \Phi\left(-\frac{(\varepsilon+\sigma^2\lambda)}{\sigma}\right) - \Phi\left(-\frac{(\varepsilon+\sigma^2\lambda(\alpha+1))}{\sigma}\right)e^C$, and $C = \varepsilon\lambda\alpha + .5\sigma^2\lambda^2(\alpha^2 + 2\alpha)$

To get the gradients, we need to differentiate the loglikelihood function with respect to each parameter. Therefore, we have

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \alpha} = \frac{1}{\alpha+1} - \frac{1}{\alpha} + \frac{1}{\mathrm{K}}\frac{\partial \mathrm{K}}{\partial \alpha}$$

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \lambda} = \frac{1}{\lambda} + \varepsilon + \lambda\sigma^2 + \frac{1}{\mathrm{K}}\frac{\partial \mathrm{K}}{\partial \lambda}$$

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \sigma} = \lambda^2\sigma + \frac{1}{\mathrm{K}}\frac{\partial \mathrm{K}}{\partial \sigma}$$

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \beta} = X\lambda + \frac{1}{\mathrm{K}}\frac{\partial \mathrm{K}}{\partial \beta}$$

The hessian matrix is the second order derivative of the log-likelihood function, and it is given by

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \alpha\alpha} = -\frac{1}{(\alpha+1)^2} + \frac{1}{\alpha^2} + \frac{\frac{\partial^2 \mathrm{K}}{\partial \alpha\alpha}\mathrm{K} - \left(\frac{\partial \mathrm{K}}{\partial \alpha}\right)^2}{\mathrm{K}^2}$$

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \alpha\lambda} = \frac{\frac{\partial^2 \mathrm{K}}{\partial \alpha\lambda}\mathrm{K} - \frac{\partial \mathrm{K}}{\partial \alpha}\frac{\partial \mathrm{K}}{\partial \lambda}}{\mathrm{K}^2}$$

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \alpha\sigma} = \frac{\frac{\partial^2 \mathrm{K}}{\partial \alpha\sigma}\mathrm{K} - \frac{\partial \mathrm{K}}{\partial \alpha}\frac{\partial \mathrm{K}}{\partial \sigma}}{\mathrm{K}^2}$$

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \alpha\beta} = \frac{\frac{\partial^2 \mathrm{K}}{\partial \alpha\beta}\mathrm{K} - \frac{\partial \mathrm{K}}{\partial \alpha}\frac{\partial \mathrm{K}}{\partial \beta}}{\mathrm{K}^2}$$

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \lambda\lambda} = -\frac{1}{\lambda^2} + \sigma^2 + \frac{\frac{\partial^2 \mathrm{K}}{\partial \lambda\lambda}\mathrm{K} - \frac{\partial \mathrm{K}}{\partial \lambda}\frac{\partial \mathrm{K}}{\partial \lambda}}{\mathrm{K}^2}$$

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \lambda\sigma} = 2\sigma\lambda + \frac{\frac{\partial^2 \mathrm{K}}{\partial \lambda\sigma}\mathrm{K} - \frac{\partial \mathrm{K}}{\partial \lambda}\frac{\partial \mathrm{K}}{\partial \sigma}}{\mathrm{K}^2}$$

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \lambda\beta} = -X + \frac{\frac{\partial^2 \mathrm{K}}{\partial \lambda\beta}\mathrm{K} - \frac{\partial \mathrm{K}}{\partial \lambda}\frac{\partial \mathrm{K}}{\partial \beta}}{\mathrm{K}^2}$$

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \sigma\sigma} = \lambda^2 + \frac{\frac{\partial^2 \mathrm{K}}{\partial \sigma\sigma}\mathrm{K} - \frac{\partial \mathrm{K}}{\partial \sigma}\frac{\partial \mathrm{K}}{\partial \sigma}}{\mathrm{K}^2}$$

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \sigma\beta} = \frac{\frac{\partial^2 \mathrm{K}}{\partial \sigma\beta}\mathrm{K} - \frac{\partial \mathrm{K}}{\partial \sigma}\frac{\partial \mathrm{K}}{\partial \beta}}{\mathrm{K}^2}$$

$$\frac{\partial \log L(\alpha,\lambda,\sigma \mid \varepsilon)}{\partial \beta\beta} = \frac{\frac{\partial^2 \mathrm{K}}{\partial \beta\beta}\mathrm{K} - \frac{\partial \mathrm{K}}{\partial \beta}\frac{\partial \mathrm{K}}{\partial \beta}}{\mathrm{K}^2}$$

Monte Carlo (MC) Simulation Study

In this section we use Monte Carlo (MC) study to examine the finite sample properties of the maximum likelihood estimator obtained from a normal-weighted exponential stochastic frontier model. A comparison is made between the maximum likelihood estimator of a normal-exponential stochastic frontier model and the maximum likelihood estimator of a normal-weighted exponential stochastic frontier model. To simulate artificial data, we have the following data generating process (DGP) of stochastic
frontier model,

$y = Xb + v - u,$

where X is $N\ by$ 3 matrix of inputs, v is the statistical error and follows the normal distribution. And the inefficiency score, u, follows a weighted exponential distribution. Artificial data on explanatory variables, $x's$, are derived from a standard uniform distribution, using built in functions in $MATLAB$. The parameter values, of the coefficients of the production function, needed for generating random output vector y are

$$b = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}.$$

The pseudo-random numbers for v are from a normal distribution with mean μ= 0 and standard deviation σ= 0.25. i.e., v~N (0,0.25). The inefficiency score, u is generated from a weighted exponential distribution. Depending on shape parameter of the weighted exponential distribution, we use two data generating process (DGP).

In the first part we have generated pseudo random numbers from the weighted exponential distribution with shape parameter α= 1 and scale parameter λ= 0.5. i.e., u~WED (1,0.5). In the second part of simulation study, we used a weighted exponential distribution with the shape parameter of α= 0.5 and scale parameter λ= 0.5, i.e., u~WED (0.5,0.5). There is no built-in function in $MATLAB$ for generating pseudo random numbers from the weighted exponential distribution. However, we can generate two independent random samples from the exponential distributions $u1$~$Exp(\lambda)$ and $u2$~Exp (λ (α+ 1)), and we add these samples to make them samples from the weighted exponential distribution, that is u= $u1$ + $u2$~WED (α, λ) (Farahani & Khorram, 1994).

In the simulation study, we have used to sample sizes of N= 500 and N= 1000. And each simulation is iterated two hundred times, (simu=200). Table 4.1. and Table 4.2. shows a Monte Carlo (MC) simulation study of the maximum likelihood estimator for a normal-weighted exponential stochastic frontier model and a normal-exponential stochastic frontier model. In the first column, the parameters of composite error term and parameter of production function are listed, and the corresponding true parameter values are in the second column. As the simulation result shows, the maximum likelihood estimates of the normal-weighted exponential stochastic frontier model are not far from the true parameter values.

When the sample size is 500, the average estimate for the shape parameter is 1.1047 and the bias is ($1 - 1.1047 = -0.1047$). But if we increase the sample size to 1000 the average estimate for the shape parameter becomes 0.9812 and the bias decreases to 0.0188. Similarly, when the sample size increases, the standard error of estimates for the shape parameter decreases from 0.9421 to 0.6235. Because there is no shape parameter in a normal-exponential stochastic frontier model, there is no estimate for the shape parameter of α= 1. For the rest of the parameters the average estimate and the standard errors, under the two stochastic frontier models, are reported in the Table 1 below.

The simulation study shows the superior performance of a normal-weighted exponential stochastic frontier model over a normal-exponential stochastic frontier model. A very significant difference is in estimating the intercept of the production function. In our simulation study, the true value of an intercept in the data generating process (DGP) is 3 and the corresponding estimate under a normal-weighted exponential stochastic frontier model is 2.9806, which means the bias is 0.0194. However, under a normal-exponential stochastic frontier model the intercept estimate is 2.3144, and the bias is 0.6856. If we increase the sample size, when N= 1000, the bias of estimating the intercept, in a normal-weighted exponential stochastic frontier model, is 0.0049, but for a normal-exponential the bias is 0.6907.

Table 1

Simulation of A Normal-Weighted Exponential Stochastic Frontier Model (When Alpha Is One)

Parameters	True Values	N=500				N=1000			
		Weighted Exponential		Exponential		Weighted Exponential		Exponential	
		Est.	Std. Err.	Est.	Std. Err.	Est.	Std. Err.	Est.	Std. Err.
α	1	1.1047	0.9421	-	-	0.9812	0.6235	-	-
λ	0.5	0.5243	0.0795	0.4347	0.0272	0.5203	0.0591	0.4330	0.0193
σ	0.25	0.1918	0.1298	0.5174	0.0854	0.2278	0.0767	0.5270	0.0612
$Cons.$	3	2.9806	0.1758	2.3144	0.1733	2.9951	0.116	2.3093	0.1256
b_1	2	2.0177	0.1943	2.0028	0.1955	2.0039	0.1386	2.0077	0.1462
b_2	1	1.0145	0.1918	1.0095	0.1992	1.0049	0.1262	1.0063	0.1311

***Note: Est. stand for parameter estimates, while Std. Err. is the corresponding standard error

Source: Own estimation

In the second part of our simulation study, we have generated artificial data for the inefficiency error distribution of a weighted exponential distribution with a shape parameter $\alpha = 0.5$. As shown in Table 2., the simulated maximum likelihood estimates of the normal-weighted exponential stochastic frontier are satisfactory. For the sample size of 500, the biased of estimating the shape parameter α is 0.197. When the sample size increases to 1000 the biased of estimating the shape parameter decreases to 0.0146. Similarly, other parameters are estimated with a small bias. When the sample size increased from 500 to 1000 the bias also decreased substantially.

The second simulation study shows the better performance of the normal-weighted exponential stochastic frontier model over the normal-exponential stochastic frontier model. For example, under a normal-weighted exponential stochastic frontier model, the bias in estimating the intercept of the production function is 0.0351. However, under a normal-exponential stochastic frontier model, the bias of estimating the intercept is 0.8263.

Table 2

Simulation of a Normal-Weighted Exponential Stochastic Frontier Model (When Alpha Is 0.5)

Source

Parameters	True Values	N=500				N=1000			
		Weighted Exponential		Exponential		Weighted Exponential		Exponential	
		Est.	Std. Err.	Est.	Std. Err.	Est.	Std. Err.	Est.	Std. Err.
α	0.5	0.6970	0.6286			0.5146	0.4854		
λ	0.5	0.5069	0.0757	0.4014	0.0257	0.5193	0.0642	0.4001	0.0185
σ	0.25	0.1923	0.1342	0.5976	0.0988	0.2254	0.0855	0.6089	0.0715
$Cons.$	3	2.9649	0.1866	2.1737	0.1991	2.9898	0.1260	2.1666	0.1469
b_1	2	2.0175	0.2136	2.0028	0.2191	2.0032	0.1517	2.0080	0.1658
b_2	1	1.0218	0.2181	1.0088	0.2229	1.0053	0.1385	1.0071	0.1483

Source: Own estimation

Estimating Carbon Efficiency of Manufacturing Firms in Africa

Data on African manufacturing firms is provided by the World Bank Enterprise Survey (WBES) data, and it is accessible online, using the link https://www.enterprisesurveys.org/. In estimating the carbon efficiency of manufacturing firms in Africa, the dependent variable is fuel consumption of firms. And the World Bank's Enterprise Survey (WBES) data set contains information about firms' fuel consumption. Moreover, the WBES data contains information about the level of sales, which is used as proxy for output. Also note that, the sale variable as proxy of output has also been used by World Bank (2011) study on productivity, using same enterprise survey data. Moreover, the enterprise survey data also includes information about labor, and it is measured by total compensation of workers including wages, salaries, and bonus. Capital is measured by replacement value of Machinery, vehicles, and equipment. Another independent variable is the intermediate good and it is measured by the cost of raw materials and intermediate materials.

In estimating carbon efficiency of manufacturing firms, we have estimated the input requirement function. Fuel consumption is the dependent variable, and the independent variables are firm's output, labor, capital, and intermediate inputs. We have estimated stochastic frontier models and three different probability distributions are assumed for the inefficiency part. As it shown in the Table 3. below all the input variables and output are statistically significant in explaining variations in fuel consumptions. In all three stochastic frontiers model specifications the estimated coefficients are almost the same and all are statistically significant at 1% level of significance. However, in a normal-half normal stochastic frontier model the intercept and the parameter of the inefficiency distribution are not statistically significant. In the normal-exponential and normal-weighted exponential stochastic frontier models all the estimates are statistically significant. Fuel consumption is positive related with output and other factors of production.

Table 3

Simulation of Fuel Consumption Under Different SFES

Variables	Stochastic Frontier Models		
	Half-normal	Exponential	Weighted Exponential
ln_Output	0.239 ***	0.239 ***	0.238 ***
	(0.025)	(0.018)	(0.001)
ln_Labor	0.371 ***	0.371 ***	0.371 ***
	(0.025)	(0.019)	(0.001)
ln_Capital	0.099 ***	0.099 ***	0.099 ***
	(0.015)	(0.010)	(0.002)
ln_Intermediate	0.169 ***	0.169 ***	0.169 ***
	(0.025)	(0.022)	(0.003)
_cons	-1.055	-1.105 ***	-1.229 ***
	(3.140)	(0.034)	(0.002)
σ_v	1.735 ***	1.734 ***	1.729 ***
	(0.028)	(0.025)	(0.0003)
σ_u	0.002		
	(3.927)		
λ		18.623 ***	7.013 ***
		(1.209)	(0.001)
α			1.965 ***
			(0.002)
Log likelihood	3764.4058	3764.4	3753.4
Number of Observation	1,911	1,911	1,911

Standard errors in parentheses
$^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

Descriptive Statistics of Carbon Inefficiency Estimates

The primary interest in fitting a Stochastic Frontier Model is to obtain the estimates of carbon inefficiency score for each manufacturing firms. Carbon inefficiency is the ratio of actual fuel consumption to the frontier (minimum) fuel consumption. Since the actual fuel consumption is always greater than the optimal fuel consumption the inefficiency estimates are always greater than one. Therefore, if a firm's fuel consumption is equal to the minimum possible frontier fuel consumption level, then the firm is fully efficient. However, if a firm consume higher than the minimum required, by the frontier function, then we have inefficiency. Alternatively, we can use another representation of carbon efficiency. Carbon efficiency level is the exponent of negative of value of the inefficiency score. And the inefficiency scores estimates are the values that we get from a composite error term.

Table 4 shows an average, minimum, and maximum values of carbon inefficiency estimate across different stochastic frontier models. The estimates in the parenthesis are the corresponding carbon efficiency level. All manufacturing firms have an estimate of efficiency level which is close to one, which is a super efficiency estimate. If we estimate using a normal-half normal stochastic frontier model, we get carbon efficiency between 99.897% and 99.898%. Which indicates that all manufacturing firms in Africa are supper efficient in their full consumption. Similarly, if we use a normal-exponential stochastic frontier model, the estimates of carbon efficiency are between 96.312% and 97.509%. For a normal-weighted exponential stochastic frontier model the minimum carbon efficiency estimate is 91.16% and the maximum carbon efficiency estimate is 95.09%.

Table 4

Descriptive Statistics of Carbon Inefficiency Estimates

	Mean	Minimum	Maximum
Half-normal	1.002354 (0.998979)	1.002344 (0.998984)	1.002362 (0.998976)
Exponential	1.074752 (0.969177)	1.059799 (0.975092)	1.090364 (0.963126)
Weighted Exponential	1.15506 (0.939315)	1.122895 (0.950907)	1.237519 (0.911602)

Source: Own Estimation

Rank of African Countries Based on Carbon Efficiency

In our study of the carbon efficiency of manufacturing firms in Africa, we have ranked African countries based on their carbon efficiency. We have efficiency estimates under three distribution assumptions for the inefficiency error in the stochastic frontier model. Table 5. shows that in all different model specifications, Egypt is the most carbon efficient country. Egypt has carbon efficiency of 99.89% under the normal-half normal stochastic frontier model, 96.99% under a normal-exponential stochastic frontier model, and 94.21% under a normal-weighted exponential stochastic frontier model. The rank of three most efficiency countries remain the same regardless of different specification of the inefficiency error.

Table 5

Rank of African Countries Based on Carbon Efficiency

	Half-normal		Exponential		Weighted Exponential	
Rank	Countries	Efficiency	Countries	Efficiency	Countries	Efficiency
1	Egypt	0.998979632	Egypt	0.969897048	Egypt	0.942098777
2	Morocco	0.998979447	Morocco	0.969595461	Morocco	0.941066612
3	Ghana	0.998979397	Ghana	0.969546412	Ghana	0.940978019
4	Ethiopia	0.998979333	Ethiopia	0.969430163	Botswana	0.94037868
5	Tunisia	0.99897931	Tunisia	0.969391918	Tunisia	0.940375189
6	Botswana	0.998979285	Botswana	0.969390826	Ethiopia	0.94020189
7	Angola	0.998979177	Burkina	0.969171487	Burkina	0.939469457
8	Burkina	0.998979167	Angola	0.969148905	Angola	0.939353536
9	Burundi	0.998978972	Burundi	0.968828249	Burundi	0.938101772
10	Zambia	0.998978941	Zambia	0.968745477	Tanzania	0.937792434
11	Nigeria	0.998978894	Tanzania	0.968737156	Djibouti	0.937787973
12	Tanzania	0.998978888	Nigeria	0.968716511	Madagascar	0.937497806
13	Madagascar	0.998978866	Djibouti	0.96871447	Nigeria	0.937467785
14	Malawi	0.998978786	Madagascar	0.968671356	Zambia	0.93733418
15	South Sudan	0.998978761	Malawi	0.968537718	Malawi	0.937004853
16	Djibouti	0.998978744	South Sudan	0.96845909	South Sudan	0.936802902
17	Uganda	0.998978699	Uganda	0.968408457	Uganda	0.936035393
18	Mauritania	0.998978628	Mauritania	0.9682597	Mauritania	0.935745438

Determinants of Carbon Efficiency of Manufacturing Firms in Africa

After estimating the carbon efficiency level of each manufacturing firm, it is also possible to estimate the determinants of carbon efficiency of manufacturing firms in Africa. To estimate the determinants of a carbon efficiency of manufacturing firms we have used the Ordinary Least Square (OLS) method. The efficiency estimate derived from the stochastic frontier model is the dependent variable and the independent variables are managerial experience, financial obstacle, firm size, import or export status, and foreign ownership. We have run the regression model on the efficiency estimates derived from three stochastic frontier models. These stochastic frontier models are normal-half normal, normal-exponential, and normal-weighted exponential.

Here we are estimating the determinants of carbon efficiency of manufacturing firms in Africa for two reasons. One is to compare the performance of the normal-weighted stochastic frontier model with other stochastic frontier models. The second reason is to examine the derivers of carbon efficiency of manufacturing firms in Africa. Table 4.7. shows that most of the variables included the regression equation are statistically significant. Comparing the three stochastic frontier models, most variables are statistically significant and $R2$ is higher under the normal-weighted exponential stochastic frontier model. Therefore, of the three stochastic frontier models the normal-weighted exponential stochastic frontier model explains most of the variations in the efficiency level of manufacturing firms in Africa. There exists a copious of studies in estimating the determinant of efficiency level of manufacturing firms (Smriti & Khan, 2018). Smriti and Khan (2018) found that the firm size, manager's experience, and annual losses due to power outage are important variable in explaining why some firms are more efficient than others. In addition, foreign ownership and exporting status are important variable in explaining the productivity difference in manufacturing sector (Islam & Hyland, 2018).

Even though most of the variables are statistically significant, their marginal effect is small, and all independent variables poorly explain the overall variations in the efficiency level. Managerial experience is a continues variable and it measures the number of years that the CEO or top manager has served the company. As it is shown in the Table 4.7, in all three stochastic frontier models, managerial experience negatively affects carbon efficiency of the manufacturing firms. However, the marginal effects are very small, for the normal-half normal stochastic frontier model a one-year increase in managerial experience increases the carbon efficiency by$2.95\text{e} - 08$. And the marginal effect of managerial experience in normal-exponential and normal-weighted exponential models are 0.0000510 and 0.000218, respectively.

Table 6

Determinants of Carbon Inefficiency

Independent Variables	Models Half normal	Exponential	Weighted Exponential
Manager Experience	-2.95e-08***	-0.0000510***	-0.000218***
	(3.83e-09)	(0.00000662)	(0.0000272)
Financial Obstacle			
Minor Obstacle	0.000000440***	0.000789***	0.00366***
	(0.000000126)	(0.000218)	(0.000899)
Moderate Obstacle	0.000000218	0.000375	0.00149
	(0.000000125)	(0.000217)	(0.000891)
Major Obstacle	0.000000372**	0.000639**	0.00241**
	(0.000000130)	(0.000225)	(0.000925)
Very Sever Obstacle	0.000000263	0.000441	0.00155
	(0.000000146)	(0.000253)	(0.00104)
Firm Size			
Small	-0.000000767	-0.00138	-0.00560
	(0.000000590)	(0.00102)	(0.00418)
Medium	-0.00000111	-0.00198	-0.00824*
	(0.000000591)	(0.00102)	(0.00418)
Large	-0.00000149*	-0.00261*	-0.0104*
	(0.000000592)	(0.00102)	(0.00419)
Export	8.95e-18*	1.62e-14*	7.14e-14*
	(4.06e-18)	(7.02e-15)	(2.88e-14)
Foreign	3.38e-09*	0.00000598*	0.0000267**
	(1.39e-09)	(0.00000240)	(0.00000990)
_cons	1.002***	1.077***	1.164***
	(0.000000591)	(0.00102)	(0.00419)
N	1759	1759	1745
R^2	0.086	0.086	0.089

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: Own Computation

Conclusion and Recommendation

In this study a weighted exponential distribution is used as the distribution of the inefficiency score in a stochastic frontier model. A weighted exponential distribution is a flexible two parameter distribution, and it is possible to derive a closed form likelihood function and JLMS inefficiency estimator for a normal-weighted exponential stochastic frontier model. Moreover, we have derived the gradient and hessian matrix of the likelihood function of a normal-weighted exponential stochastic frontier model.

A Monte Carlo (MC) simulation study is implemented to examine the finite sample properties of the maximum likelihood estimator of a normal-weighted exponential stochastic frontier model. Pseudo random numbers are generated using built-in functions for generating random numbers in $MATLAB$. A comparison is made with maximum likelihood estimator of a normal-exponential stochastic frontier model. The simulation result shows that, a normal-weighted exponential stochastic frontier model performs well compared to a normal-exponential stochastic frontier model, given the data generating process is a normal-weighted exponential stochastic frontier model. As the sample size increases the bias and the standard errors of the maximum likelihood estimator of a normal-weighted exponential distribution decrease.

To demonstrate the usefulness of the new stochastic frontier model, a normal-weighted exponential stochastic frontier model, we have used a real data application. The real data application is on carbon efficiency of manufacturing firm in Africa. Three stochastic frontier models are estimated to get the carbon efficiency estimates of each manufacturing firms in Africa. All the three stochastic frontier models give almost similar estimates for the parameters of a production function. A summary of descriptive statistics of carbon efficiency estimates are discussed. Moreover, African countries will be ranked based on their carbon efficiency level in their manufacturing sector. Our estimation result shows that of the 18 African countries covered in the study, Egypt it top one carbon efficiency country. Moreover, we have also estimated a multiple linear regression model to see the determinants of carbon efficiency of manufacturing firms in Africa. Top managers experience in the firm, the degree of obstacle for financial access, firm size, export, and foreign ownership are important variables explaining variations in carbon efficiency of manufacturing firms in Africa.

Based our studies we recommend the following, both for researchers and policy makers. The likelihood function of a normal-weighted exponential stochastic frontier model is flexible model, and it provides a closed form solution. Therefore, applied researchers are recommended to use it for their productivity and efficiency analysis. Our derivation of the likelihood function is for cross sectional data. Therefore, a researcher is recommended to extend the model for times series and panel data models.

Researchers interested on Monte Carlo (MC) simulation study can extend the study into times series and panel data cases. Moreover, it is also possible to compare other stochastic frontier models with a normal-weighted exponential stochastic frontier model.

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