Exploring Pre-service Mathematics Teachers’ Beliefs: An Evaluative Study

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Abstract: This study examines the mathematical beliefs of pre-service mathematics teachers in Mekelle College of Teacher Education. One hundred and eighty college mathematics majoring students were drawn from first year, second year, and third year batches in the 10+3 diploma program in the College. A mathematical belief instrument (MBI) was used to collect data for the research. To analyze the data, discrete statistics/percentage and chi-square statistics were used. The finding of the study indicated that student beliefs across the themes were inconsistent with the current recommendations and philosophy of mathematics teaching and learning in general. The study also noted that the pre-service teachers’ college year (i.e. their being 1st, 2nd or 3rd year) had no relationship with their mathematical belief. However, a relationship was noted between the trainees’ years in the college and their self-efficacy.

Introduction

Students’ beliefs about mathematics learning and teachers’ beliefs about mathematics teaching have been points of discussion for a long time. In particular, over the last two decades, research reports have provided evidence about students’ negative beliefs towards the learning, teaching and the nature of mathematics (Hart, 2002; Hoffmann, 2001; Schoenfeld, 1992; McLeod, 1992; Emenaker, 1996). In this connection, Wong et al. (2002), affirm that beliefs about mathematics learning, beliefs about mathematics teaching, and beliefs about the self-situated learning in a social context in which mathematics is taught and learned are closely related to the students’ motivation to learn and their performance in the subject. Students’ failure to

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solve mathematical problems is directly attributable to their powerful beliefs about the nature of mathematics and problem solving (Schoenfeld, 1992).

Beliefs held by students could significantly impact their learning of mathematics. Positive beliefs imply students’ success in learning of mathematics while negative beliefs may result in their low self-confidence, low self-esteem, low self-perception and learned helplessness (Renga and Dalla, 1993; Hart and Walker, 1993; Boaler, 1997).

At the post primary level, Conway and Sloane (2005) as well as Cangelosi (1996) investigated that student negative beliefs result in undesirable effects including viewing mathematics: (1) as a boring sequence of technical vocabulary, rules, and algorithms to be memorized for the purpose of passing tests; (2) as a male domain subject; (3) as peculiar to individuals with an exceptional aptitude and creativity; and (4) as a complex, mystifying subject that was handed down by ancient mystics such as Greek mythology. This study, therefore, was designed to investigate the types of dominant beliefs pre-service student teachers held towards mathematics learning, teaching, and its nature.

**Statement of the Problem**

The primary goal of mathematics instruction is to help students in building up the belief that they can do mathematics on their own endeavor. According to Renga and Dalla (1993), students’ beliefs, feelings, and perceptions are related to their confidence in learning mathematics and mathematics anxiety.

However, several studies reported that students learning mathematics are encumbered with beliefs that are incompatible with current philosophies and classroom recommendations (Conway and Sloane, 2005; Madden, 2008; Stevens, 2005). The logical question, then, would be: “If such a problem is observed broadly among learners majoring mathematics globally, can we expect a different observation among college students learning mathematics in our context?”
This study investigated the prominent limiting beliefs that college mathematics pre-service teachers held in relation to the teaching and learning of mathematics. Particularly, the study attempted to seek answers to the following questions:

- What are the prominent beliefs demonstrated by college mathematics pre-service teachers about the nature of mathematics, mathematics learning, and mathematics teaching?
- To what extent are these beliefs compatible with current philosophies and recommendations of mathematics learning and teaching?
- What is the level of pre-service mathematics teachers’ confidence about mathematics teaching and learning?

Research Hypotheses

The following hypotheses were tested in the research project.

- There will be momentous relationship between the themes of beliefs held by pre-service mathematics teachers the number of years they spend learning in the college.
- There would be significant relationship between the year pre-service mathematics teachers spent learning mathematics and their beliefs about mathematics teaching and learning.
- There would be relevant relationship between the year mathematics pre-service teachers were attending and their confidence in mathematics teaching and learning.

Definition of Operational Terms

Belief Systems- belief systems often embrace affective feelings and evaluation, vibrant recollections of personal experiences, and assumptions about the existence of entries and alternative words (Thompson, 1992).
**Self-confidence** - “confidence in learning mathematics has to do with how sure a person is being able to perform well in mathematics, learn new topics in mathematics, do well on mathematics tests, and solve none-routine problems” (Hart and Walker, 1993, p.24).

**Self-efficacy** - “Self-efficacy is linked with the belief an individual has about his/her ability to perform in a given situation-students’ beliefs in themselves and their ability to learn.” It is connected to decisions about which activities the student chooses to participate in, how much effort they spend, and how long she/he keeps on these activities (McLeod, 1992; Fairbrother, 2000, p.17).

**Review of Related Literature**

**Foundation of Current Philosophies and Recommendations of Mathematics Instruction**

Much time has been spent in describing the complex nature of teaching and learning since the time of Socrates. It has been argued that these acts are complex and need to be understood as an interaction of various kinds of knowledge resources and practices (Brodie, 2001). Philosophers and psychologists have worked for centuries to analyze these acts and they have come up with various learning theories that attempt to describe the process (Pollard, 2002). Two theories have had a particular influence on teaching and learning (Biggs, 1996; Black, 1999; Cobb, 1999). The first is *behaviorism*, which advocates rule-bound traditional rote learning as well as content- and teacher-dominated teaching which places the focus on “transmission of information.” Indeed, this has much in common with the traditional teaching approaches (Armour -Thomas and Allen, 1993). However, traditional teaching approaches that discourage students’ engagement mislead students to develop wrong beliefs of mathematics as a school subject packed with procedures, rules, and algorithms to be memorized (Keegan, 1995). Wrong beliefs about the nature of mathematics and problem solving lead to students’ failure in solving mathematical
problems (Schoenfeld, 1992; Mac an Bhaird, 2009). The second is referred to as \textit{constructivism}, which promotes a more learner-oriented approach to learning and open methods through which learners recognize the applications of mathematics in their daily lives (Boaler, 1997; Cobb et al, 1991; Principles and Standards for School Mathematics (NCTM), 2000).

Pertinent to the constructivist philosophy is underpinned by the belief that the learners are active and innovative individuals, having their own interest in and capacity for knowledge and self-development (Felder & Brent, 2002; Leu, 2002), blending specific instructional strategies such as problem solving, cooperation, inquiry, discovery, and meta-cognitive strategies in the learning and teaching of mathematics (Harris et al, 2001). The constructivist instructional model is frequently designed to expose learners to a learning environment or material where they can make observations conflicting with their own beliefs and experiences (which are usually wrong or incomplete) or unfamiliar at all. For the constructivists, when learners are exposed to beliefs, experiences, and/or observations conflicting with their own, they would be motivated to settle the conflict. In this regard, many constructivist scholars recommend the use of problematic situations to enable students to construct their own knowledge by means of 'doing' mathematics, solving problems and organizing the subject matter (Dossey, 1992; Freudenthal, 1971; Polya, 1988) and thereby challenge the limiting beliefs. To achieve higher order cognitive goals of the type, courses should be built around problems in such a way that students can spend much of their time discussing problems in groups instead of constantly waiting for the teacher to explain it to them. Moreover, akin to canonical understandings and scientific ideas, historical facts and developments of events can serve as sources of cognitive growth to challenge long-held wrong beliefs. In such cases, complex mathematical tasks chosen carefully by the teacher will help “an individual’s ability to explore, conjecture, and reason logically . . . to use a variety of mathematical methods effectively to solve non-routine problems” (NCTM, 1995, p.5). Hiebert and his colleagues termed this as problematizing mathematics. To this end, “…problematic situations help learners to think critically about why things are true, to inquire, to search for solutions, and to
resolve incongruities” (Hiebert et al, 1999, p.151). This, according to Van de Walle (1998, p.483), involves three fundamental components: (1) a task, a situation, or problem presented to the students for consideration; (2) an opportunity for students to work on the task; and (3) an opportunity for discussion and reflection on the work done by the students after accomplishing the task. This help learners challenge constraining beliefs and demonstrate canonical/positive beliefs.

The lack of correct belief systems among adult learners; e.g. wrong belief systems about the nature of mathematics among college students, cannot be regarded as purely cognitive behavior. It is rather influenced by the belief systems the learners held. Hence, availability of correct belief systems and treatments that lead to the development of correct beliefs promote learning (Goulding, 2004; Beswick, 2007; Mac an Bhaird, 2009; Rogoff, 1999). An instructional material that connects mathematics lessons with prior knowledge and related concepts and demonstrates how the concepts are applied challenges wrong beliefs and promotes learning (Boaler, 1997).

**Beliefs and their Impact on Mathematics Learning and Teaching**

The most important goal of mathematics tuition is to help students in building up the belief that they can do mathematics on their own endeavor. Renga and Dalla (1993) point out that the strong bond among students’ beliefs, feelings, and perceptions are uncovered in relation to confidence in learning mathematics, mathematics anxiety, perceptions of the causes of success and failure, and learned helplessness. However, confidence in learning mathematics and self-efficacy in mathematics are strongly related beliefs that pertain to whether students think that they can learn new mathematics concepts or ideas and achieve well in mathematics which positively correlate with their mathematics achievement and their ability in solving nonroutine problems(Renga and Dalla, 1993; Hart and Walker, 1993). That means students’ belief systems can influence either positively or negatively the self-confidence (or self-efficacy) of a student in mathematics learning the role of which will be explicit in subsequent paragraphs.
In general, the way one perceives mathematics matters the way s/he learns or s/he teaches (Emenaker, 1996). For example, teacher beliefs on mathematics learning and instruction appear to have direct impact on students' beliefs on learning of mathematics and its nature. While teachers with positive mathematical attitudes create conducive learning environment and thereby assist their students establish positive beliefs and attitudes towards mathematics learning and instruction, teachers with negative attitudes and beliefs toward mathematics instruction expose their students to develop undesirable effects in the learning of mathematics. In this case, while the former teacher style helps students discover relationships, construct concepts, and explore mathematical ideas, the teacher/s with the latter view encouraged learned helplessness; develop math anxiety, and low confidence in learning mathematics (Hart, 2002). As the negative beliefs have long run undesirable impacts on student learning, efforts should be made to bring about tremendous changes toward the desired effects with the teacher training institutions playing a leading role. In this regard, the following quote from Hart (2002, p.4) is a good guide toward the possible solution to the issue:

... it seems imperative that teacher education programs assess their effectiveness, at least in part, on how well they nurture beliefs that are consistent with the program's philosophy of learning and teaching. . . . they need to study how consistent the belief teachers espouse after participating in a program with their teaching practices, i.e., can teachers do more than “talk the talk.”

Categories of Beliefs

As to Hoffmann (2001), an individual’s belief system can be numerically uncountable; however, there is a simple category of the major beliefs manifested frequently. Hoffmann (2001), Hart (2002), Emenaker (1996), Boaler (1997), Simon and Schifter (1993) and others identify several traditional student beliefs as themes the summary of which will be described in this section. Perhaps, these themes can be categorized under the
headings: (1) beliefs in mathematics which comprises of the first 13 themes; and (2) beliefs in mathematics learning and teaching which include four major themes (themes 14, 15, 16 and 17).

**Theme 1: Time.** Students believe that if a math problem takes more than 5-10 minutes, it is impossible to solve (Emenaker, 1996; Hart, 2002).

**Theme 2: Step.** Students think that all mathematics problems can be solved using a step-by-step algorithm or a single equation (Emenaker, 1996; Hoffmann, 2001).

**Theme 3: Memory.** The students feel that memorizing is more valuable to learning mathematics than thinking for oneself. That means students seem to imagine mathematics is mostly memorizing facts, procedures, and rules (Schoenfeld, 1992).

**Theme 4: Understanding.** Many students believe that only the gifted few/geniuses are capable of doing or understanding formulas and equations in mathematics and even able in solving perplexing problems (Hoffmann, 2001; Wong et al, 2002).

**Theme 5: Knowing.** For most if not few, knowing mathematics is being able to get the right answer quickly (Emenaker, 1996; Wong et al., 2002).

**Theme 6: Nature.** Students view mathematics as computation using rules, procedures, and algorithms instead of giving meaning to a situation. Moreover, for most mathematics is considered as a subject which is formula laden and boring (Boaler, 1997; Lampert, 1990).

**Theme 7: Uniqueness.** Students seem to believe that there is always one correct/valid answer for a mathematical problem. They fail to recognize the existence of problems having more than one valid answer (Hoffman, 2001; Wong et al., 2002).
**Theme 8: Truth.** Several students related mathematics with certainty. In this regard truth/correctness of a mathematical problem is determined when the answer is ratified by the teacher or a textbook author (Wong et al., 2002).

**Theme 9: Solution Methods.** Students view that there is always one correct way and rule that could be followed to solve a given mathematical problem (Conway and Sloane, 2005).

**Theme 10: Discovery.** Many students adhered to the thought that mathematics is a finished product and thereby everything about important mathematics is already known by mathematicians. They also persist that average people cannot discover mathematics for themselves (Emenaker, 1996; Davis, 1996).

**Theme 11: Richness.** The student imagines mathematics as a subject that is not rich and varied (Boaler, 1997).

**Theme 12: Equity.** Students believe that mathematics is a male domain. Both male and female students deem that female students cannot perform mathematics well unless they exert an extra effort as compared with their male partners (van de Walle, 1998; Cathcart et al., 2001).

**Theme 13: Perceived Usefulness/Application.** Students reflect the view that mathematics as having little application to life outside schools (Hoffman, 2001).

**Theme 14: Role of the Student.** Students think remembering and applying the correct rules are valuable when the teacher asks questions. The student views his/her role in the classroom is a passive recipient (Wong et al., 2002; Boaler, 1997). Furthermore, they believe that the goal of doing mathematics is to obtain the correct answer and their role is to receive mathematical knowledge that could be evidenced through demonstrations.
Theme 15: **Role of the Teacher.** The role of the teacher is considered as providing information for students as passive learners, ratifying students’ works as correct or wrong, lay rules for students when students intend to do mathematics and active to spoon-feed students during mathematics learning (Wong et al., 2002; Schoenfeld, 1992).

Theme 16: **Learning.** For many students, learning is about: correct performance of tasks, accumulating information, receiving information, taking in knowledge, and practicing and performing (Reece and Walker, 2003).

Theme 17: **Teaching.** For a substantial number of students, teaching is about: giving accurate information, being sequential and hierarchal, directing one-way flow, structuring the environment, and rewarding performance (Reece and Walker, 2003; Huetinck and Munshin, 2000).

**Method**

**Participants**

The data collected from 180 pre-service mathematics teachers was analysed and interpreted. All of these respondents were students in the 10+3 diploma program. They were getting trained to teach in the middle grades (grades 5-8) after graduation. The researcher focused on these groups of respondents because they will be engaged in teaching for a long time. The level they will be assigned as teachers is sensitive and has an essential impact on students’ later learning as well as perception of mathematics. Altogether, 54 students from first year, 66 students from second year and 60 students from third year were participated in the research.

**Instrument of Data Collection**

The instrument used for the study was the Mathematical Belief Instrument (MBI). This instrument determines how consistent an individual’s beliefs are with the global views and philosophies of the subject, the teaching and
learning of mathematics, and the confidence to learn and teach mathematics (e.g., The National Council of Teachers of Mathematics, Curriculum and Evaluation Standards (NCTM), 1989; The National Council of Teachers of Mathematics, Principles and Standards for School Mathematics (NCTM), 2000). The mode of response included: true (T), more true than false (MT), more false than true (MF), and false (F). The instrument has three major parts. Part A measures student teachers’ mathematical belief and has 13 main themes. Part B assesses student teachers’ beliefs about mathematics teaching, learning, role of the teacher, and role of the students. This part has four main themes. Part C of the instrument measures pre-service teachers’ self-efficacy about the learning and teaching of mathematics. Parts A and C are adapted from Hart (2002) and Part B is adapted from Simon and Schifter (1993). The instrument was pilot tested and modified several times to improve reliability. After completing the modification of the survey instrument, it was administered to the participants. The statistical software SPSS i.e., inter-alia was used to determine the reliability of the instruments. The reliability managed from 0.78 to 0.86. To ensure the validity of the instrument, it was subjected to adept personnel in statistics in the department of mathematics. The final form of the survey had 39 items in Part A, 12 items in Part B, and 6 items in Part C. Altogether, the instrument had 57 items.

Data Analysis

The data were analyzed using simple percentage, and chi-square. While the simple percentage was used to analyze the extent to which pre-service teachers’ beliefs were scattered over the scales of the three categories, the chi-square was used to analyze the degree of relationship among the restraining sets of beliefs as themes and the year pre-service mathematics teachers were attending. Moreover, the chi-square statistics was used to gauge the degree of relationship between the pre-service mathematics teachers’ self-confidence and the year they were attending.
Findings

The number of students (n=180) who participated in the study was relatively small, because of this; descriptive statistics was used to study trends across the group. The themes listed are more generally in line with the traditional view. The items of the questionnaire are consistently established with these restraining themes. Because of this, students’ high scores in the first two columns in parts A and B of the survey indicated that students’ perceptions are more consistent with the listed themes. However, higher scores (percentages) in the first two columns of Part C indicate that students’ had better position in their self-efficacy/confidence to teach and learn mathematics.

Pre-service Teachers’ Beliefs about Mathematics (Part A)

From the survey results illustrated in Table 1, a number of valid statements may be drawn. Data analysis from Table 1 reveal that none of the beliefs demonstrated by the student teachers are consistent with the current recommendations and philosophies of mathematics learning and teaching. Furthermore, the table shows that 66-94% of the data obtained supports the limiting beliefs. For instance, about 88% (sum of responses in the first two columns) of the population is in support of the inconsistent view “If a math problem takes more than 5-10 minutes, it is impossible to solve” (Theme 1). Furthermore, the students think that every mathematical problem can be solved using a step-by-step procedure or an equation (Theme 2) which they think could be mastered through memorization (Theme 3). On the other hand, a sizeable number of the pre-service mathematics teachers believe only the gifted few are capable of doing/understanding mathematics (Theme 4), whereas others believe that knowing mathematics is related with being able to do a problem rapidly and come up with a correct solution immediately (Theme 5). In this regard, the respondents seem to believe that the certainty/truth of their work is assured by the teacher or a textbook author (Theme 8).
In addition, many of the pre-service mathematics teachers tend to hold the belief that mathematics is a computation using rules, procedures, and algorithms (Theme 6). This may lead them to having the ill-perception that the procedures, rules, and algorithms help to produce unique solution to every mathematical problem (Theme 7) by means of a single solution method or strategy (Theme 9). A common experience demonstrated by the pre-service mathematics teachers is that the limiting belief- mathematics is a finished product (Theme 10) which has limited scope and little opportunity to vary (Theme 11) as well as little application to the outside world or real life situation (Theme 13). A similar result also displayed in Table 2 for the remaining theme (Theme 12). This shows that the pre-service mathematics teachers are in support of restraining positions “mathematics is a finished product”. For more details see Table 1.
Table 1: Data on student teachers’ beliefs

<table>
<thead>
<tr>
<th>Major Categories</th>
<th>Themes</th>
<th>T</th>
<th>MT</th>
<th>MF</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part A: Beliefs about Mathematics</td>
<td>Time</td>
<td>105(58%)</td>
<td>54(30%)</td>
<td>16(9%)</td>
<td>5(3%)</td>
</tr>
<tr>
<td></td>
<td>Step</td>
<td>137(79%)</td>
<td>26(14%)</td>
<td>11(6%)</td>
<td>6(3%)</td>
</tr>
<tr>
<td></td>
<td>Memory</td>
<td>130(72%)</td>
<td>24(13%)</td>
<td>13(7%)</td>
<td>13(7%)</td>
</tr>
<tr>
<td></td>
<td>Understanding</td>
<td>139(77%)</td>
<td>20(11%)</td>
<td>12(7%)</td>
<td>10(5%)</td>
</tr>
<tr>
<td></td>
<td>Knowing</td>
<td>144(80%)</td>
<td>17(9%)</td>
<td>11(6%)</td>
<td>8(5%)</td>
</tr>
<tr>
<td></td>
<td>Nature</td>
<td>67(37%)</td>
<td>52(29%)</td>
<td>35(19%)</td>
<td>26(15%)</td>
</tr>
<tr>
<td></td>
<td>Truth</td>
<td>147(82%)</td>
<td>17(9%)</td>
<td>13(7%)</td>
<td>3(2%)</td>
</tr>
<tr>
<td></td>
<td>Methods</td>
<td>154(86%)</td>
<td>14(8%)</td>
<td>8(4%)</td>
<td>4(2%)</td>
</tr>
<tr>
<td></td>
<td>Discovery</td>
<td>95(54%)</td>
<td>55(31%)</td>
<td>17(9%)</td>
<td>11(6%)</td>
</tr>
<tr>
<td></td>
<td>Richness</td>
<td>69(38%)</td>
<td>54(30%)</td>
<td>37(21%)</td>
<td>20(11%)</td>
</tr>
<tr>
<td></td>
<td>Equity</td>
<td>133(74%)</td>
<td>32(18%)</td>
<td>9(5%)</td>
<td>6(3%)</td>
</tr>
<tr>
<td></td>
<td>Application</td>
<td>101(56%)</td>
<td>58(32%)</td>
<td>16(9%)</td>
<td>5(3%)</td>
</tr>
<tr>
<td>Part B: Beliefs about mathematics learning and Teaching</td>
<td>Learning</td>
<td>62(40%)</td>
<td>65(39%)</td>
<td>25(15%)</td>
<td>11(6%)</td>
</tr>
<tr>
<td></td>
<td>Teaching</td>
<td>60(37%)</td>
<td>64(39%)</td>
<td>27(17%)</td>
<td>12(7%)</td>
</tr>
<tr>
<td></td>
<td>Role of the Student</td>
<td>64(39%)</td>
<td>68(42%)</td>
<td>18(11%)</td>
<td>13(8%)</td>
</tr>
<tr>
<td></td>
<td>Role of the Teacher</td>
<td>62(38%)</td>
<td>67(41%)</td>
<td>30(19%)</td>
<td>4(2%)</td>
</tr>
<tr>
<td>Part C: Beliefs about self/Self-efficacy</td>
<td>Efficacy in learning</td>
<td>93(52%)</td>
<td>46(25%)</td>
<td>23(13%)</td>
<td>18(10%)</td>
</tr>
<tr>
<td></td>
<td>Efficacy in teaching</td>
<td>97(54%)</td>
<td>48(27%)</td>
<td>21(11%)</td>
<td>14(8%)</td>
</tr>
</tbody>
</table>

Pre-service Teachers’ Beliefs about the Teaching and Learning of Mathematics (Part B)

As indicated in Table 1 (Part B), pre-service mathematics teachers’ views toward learning, teaching, the role of the student, and the role of the teacher are compatible with the traditional views. Pre-service mathematics teachers’ beliefs about learning and teaching is away from the view that learning is actively engaging with concepts, reflection self monitoring interactivity and co-operation. This also appears to where that teaching is always from setting challenging tasks, promoting and sharing community values and supporting and assisting reflections.

Moreover, the respondents believe that the role of the learner is accumulating information and passively listening to the teacher. They believe
the role of the teacher is providing information and ratifying students' answers as correct or wrong only (please see Table 1).

**Pre-service Teachers' Self-Confidence in the Teaching and Learning of Mathematics (Part C)**

As is depicted in Table 1 (Part C), senior pre-service mathematics teachers demonstrate high self-confidence as compared to the year 1 students. Thus, there seems to be a relationship between the self-confidence/self-efficacy of pre-service mathematics teacher and the year they are attending.

**The Year Pre-service Teachers are attending and Mathematical Beliefs: Hypothesis I**

The result of the data analysis on the relationship between the set of themes on the subject mathematics, and the pre-service teachers' year in the college is demonstrated in Table 3.

The chi-square in Table 1 shows that with six degrees of freedom, a $\chi^2$ value of 3.070 is needed for significance at the 0.80 level. However, the obtained $\chi^2$ value of 2.421 is smaller than this table value. This is not significant. This means that the differences between the expected and the observed frequencies are not beyond what would be expected by chance. This is displayed in Table 2.

**Table 2: Students’ status and beliefs in mathematics**

<table>
<thead>
<tr>
<th>Status</th>
<th>True</th>
<th>More true than false</th>
<th>More false than true</th>
<th>False</th>
<th>Total</th>
<th>$\chi^2$ Calculated</th>
<th>$\chi^2$ Critical</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year I</td>
<td>35 (35)</td>
<td>12 (12)</td>
<td>3 (4)</td>
<td>4 (3)</td>
<td>54</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year II</td>
<td>42 (42)</td>
<td>17 (15)</td>
<td>4 (5)</td>
<td>3 (4)</td>
<td>66</td>
<td>2.421</td>
<td>3.070</td>
<td>6</td>
</tr>
<tr>
<td>Year III</td>
<td>39 (39)</td>
<td>12 (14)</td>
<td>6 (4)</td>
<td>3 (3)</td>
<td>60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>116</td>
<td>41</td>
<td>13</td>
<td>10</td>
<td>180</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
As can be understood from Table 2, there is no evidence to relationship between the variables pre-service mathematical teachers’ beliefs and the year the pre-service teachers are attending. More importantly, the most popular constraining mathematical beliefs are uniformly demonstrated (or held) by all the students throughout the years. Because of this, with 6 degree of freedom, the hypothesis that “There will be momentous relationship between the themes of beliefs held by pre-service mathematics teachers’ and the year they are attending” was rejected and hence the null hypothesis “There is no momentous relationship between the themes of beliefs held on mathematics by pre-service mathematics teachers’ and the year they are attending” is was retained.

Table 3: The year pre-service teachers are attending and beliefs about the teaching and learning of mathematics

<table>
<thead>
<tr>
<th>Status</th>
<th>True</th>
<th>More true than false</th>
<th>More false than true</th>
<th>False</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year I</td>
<td>15 (19)</td>
<td>18 (20)</td>
<td>12 (8)</td>
<td>4 (3)</td>
<td>49</td>
</tr>
<tr>
<td>Year II</td>
<td>28 (23)</td>
<td>25 (24)</td>
<td>4 (9)</td>
<td>3 (4)</td>
<td>60</td>
</tr>
<tr>
<td>Year III</td>
<td>19 (21)</td>
<td>23 (22)</td>
<td>9 (8)</td>
<td>3 (3)</td>
<td>54</td>
</tr>
<tr>
<td>Total</td>
<td>62</td>
<td>66</td>
<td>25</td>
<td>10</td>
<td>163</td>
</tr>
</tbody>
</table>

Table 3 shows that there was no significant relationship between the year pre-service mathematics teachers are attending and their beliefs about the teaching and learning of mathematics. If we refer to the table of chi-square, we can see that with six degree of freedom, a $\chi^2$ value of 8.558 is needed for significance at the 0.20 level. However, the obtained $\chi^2$ value of 7.893 is smaller than this table value and is therefore not significant. This means that the difference between the expected and the observed frequencies is not beyond what would be expected by chance. Thus, the hypothesis “There will be significant relationship between the year pre-service mathematics teachers’ are attending and their beliefs toward mathematics teaching and learning” is rejected and hence the null hypothesis “There will be no significant relationship between the year pre-service teachers are attending
and their beliefs toward mathematics teaching and learning” is accepted in place.

Table 4: The year pre-service teachers spend learning mathematics in the college attending and efficacy in the teaching and learning of mathematics

<table>
<thead>
<tr>
<th>Status</th>
<th>True More true than false</th>
<th>More false than true</th>
<th>False</th>
<th>Total</th>
<th>$\chi^2$ Calculated</th>
<th>$\chi^2$ Critical</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year I</td>
<td>20(28)</td>
<td>17 (14)</td>
<td>9(7)</td>
<td>8(5)</td>
<td>54</td>
<td>15.778</td>
<td>15.033</td>
</tr>
<tr>
<td>Year II</td>
<td>32(35)</td>
<td>20 (17)</td>
<td>10(8)</td>
<td>4(6)</td>
<td>66</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>Year III</td>
<td>43(32)</td>
<td>10(16)</td>
<td>3(7)</td>
<td>4(5)</td>
<td>60</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td>95</td>
<td>47</td>
<td>22</td>
<td>16</td>
<td>180</td>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>

Table 4 indicates that there is significant relationship between pre-service mathematics teachers and their ‘self-efficacy in the teaching and learning of mathematics. At 0.02 level of confidence, the hypothesis “There will be relevant relationship between the year pre-service mathematics teachers are attending and their confidence in mathematics teaching and learning” is retained.

Discussion of Findings

The discussion of the findings in this section focuses on the pre-service teachers’ beliefs in mathematics and beliefs in the learning and teaching of mathematics as well as the dissemination and strength of beliefs over the cohorts of pre-service teachers in mathematics.

Beliefs about Mathematics

The finding displayed in Table 2 indicates that students’ beliefs about mathematics reflect a more traditional perspective on the subject as well as on the teaching and learning of mathematics.
This means that pre-service teachers were more likely to maintain the view that “a mathematics problem must be solved in a minimum possible time, otherwise it is impossible to solve.” They were also more likely to support the belief that “every mathematical problem has a unique solution.” In relation to the first conception, Emenaker (1996) found similar results and pointed out that students’ beliefs about mathematics may weaken their ability to solve non-routine problems. If students believe that mathematical problems should always be completed in ten minutes or less, they may be unwilling to persist in trying to solve problems that may take substantially longer for most students (McLeod, 1992).

Mac an Bhaird (2009) report that students in higher institutions have little opportunity to see mathematics as a continuous and growing subject where history plays a key role in this regard. Students lack the awareness that some mathematical concepts have taken thousands of years to develop to their current form. There is no chance of see the bigger picture and realize their classroom experience that involves success and failure to demonstrate mathematical understanding is not automatic. In the same way as the classroom experience of success and failure, the historical growth of mathematics was accompanied by endless ups and downs. There is ample evidence from the history of mathematics that the above student beliefs are unfounded. History shows that quite a number of mathematical ideas have remained unsolved for years and yet there are a considerable number of mathematical ideas that have not achieved solutions. For example, while Fermat’s Last Theorem remained unsolved for more than three centuries, the Four Color Conjecture (now Theorem) remained unsolved for more than 125 years. On the other side of the continuum, there are yet several conjectures such as: the Goldbach’s Conjecture, the Twin Prime Conjecture, the Odd Perfect Number Conjecture, Ulam’s Conjecture and some others (Eves, 1990) whose solutions/proofs have not been obtained.

As depicted in Table 1, the assessment of the remaining themes indicates similar results. This includes: “knowing is related with solving and getting the right answer quickly; “mathematics is by and large memorization”; “only the
geniuses are capable of doing mathematics”, “mathematics is a finished product”, “the truth for a problem is ratified by the teacher or the textbook author” “mathematics has little application outside classrooms”, and “mathematics is computation using rules, procedures, and algorithms”. This finding is supported by a number of reports. For example, in the eye of students, mathematics is regarded as a body of absolute truth (Fleener, 1996), associated with certainty (Lampert, 1990) and viewed as a set of rules with symbols for playing around (Kloosterman, 1991). Moreover, Lampert (1990) claims that students’ views about mathematics are ill-conceived in many ways. For example, doing mathematics means following rules laid down by the teacher and a textbook, knowing is associated with remembering and applying the correct rule and getting the answer quickly, and mathematical truth is determined when the answer is ratified by the teacher. In the end, such types of beliefs in mathematics influence how students and teachers perform in the elementary schools (Stodolsky, 1985).

One particular instance that needs special attention is the view “mathematics is a male domain.” In this research project, the majority of students including females believe that females need an extra effort to be as competent as their male partners in mathematics. This finding is in agreement with van de Walle’s (1998) and Cathcart et al.’s (2001) report that both male and female students persist the “maleness of mathematics” which implies the beliefs that males out perform their female associates in mathematics. Furthermore, Cathcart et al., (2001) list five major myths in the United States of America which might have some correspondence to our context. These include:

- Myth 1: Some children cannot learn math;
- Myth 2: Boys learn math better than girls;
- Myth 3: Poor children and children from underrepresented groups cannot learn math;
- Myth 4: American children have less mathematical ability than Asian children; and
- Myth 5: Mathematics learning disabilities are common.
It seems imperative that creating classroom cultures that help students challenge limiting beliefs in the learning and teaching of mathematics as well as how they nurture beliefs that are consistent with the philosophy of learning and teaching mathematics is vital.

**Beliefs about Mathematics Teaching and Learning**

Student teachers' views of the concepts of learning and teaching mathematics are in line with the traditional view that learning is passively receiving information and teaching is transmission of information from the teacher to the student. This is displayed in Table 1.

In this regard, pre-service teachers fail to view learning as the active construction of prior knowledge and the collaborative construction of socially defined knowledge and values. In fact, learning happens by means of multiple opportunities, diverse processes, and socially constructed opportunities to connect to what is already known. Furthermore, students seem to be unaware that learning can come about through independent teaching or the interdependence of the teacher and the students. In this context learning involves learning with understanding, actively building new knowledge from experience and prior knowledge. On the other hand, students lack the understanding that teaching - is challenging students, and guiding thinking towards a more complete understanding. Students should notice that teaching plays the role of providing an opportunity for them to gradually construct knowledge and skills through experience, interaction, and teacher support. Also they must be clear that teaching includes understanding what students ought to know and need to learn (Ernest, 1996; Huetinck & Munshin, 2000; NCTM, 2000; Pollard 2002).

From this discussion, it appears important to describe the student's role and the teacher's role in the teaching and learning of mathematics. Huetinck & Munshin, (2000) and Marsh, (2004) as well as Pollard, (2002) described the role of the teacher as organizer of the learning environment, assessor of students' thinking; and initiator of group activities. In the first place the
teacher acts as a facilitator, a guide, and a co-participant. In the second place the teacher plays the role of active listener. He listens to students’ conceptions, ideas, individually and socially, and co-constructs different interpretations of knowledge. Correspondingly, the role of the student is defined as active constructor, with others and with him/herself. The student is thought of as an active thinker, explainer, interpreter, questioner, and working as an individual or active social participant. He/she is motivated intrinsically and socially to construct his/her own knowledge.

Data presented in Tables 2 through 3 indicated that the limiting beliefs are held uniformly throughout the batches. This indicates that not only the efforts employed to change pre-service teachers’ beliefs toward mathematics, learning teaching mathematics is minimal, but it also show that the classroom culture and contexts are not as such effective in challenging the restraining beliefs held by students through their schooling time as well as during their stay in the teacher education colleges. It seems important to note that the findings reported above are the consequences of teacher classroom actions and cultures which also in turn tremendously impact students’ belief systems. Schoenfeld (1992) and Emenaker (1993) indicate that teachers have a momentous role in students’ achievement, and their formulation of beliefs and attitudes toward mathematics. Hart (2002, p.4) maintains that “beliefs are created through experience over time, pedagogical practices that support constructivist theory can be nurtured by engaging novice teachers in constructivist experiences both in learning mathematics and in teaching mathematics”.

**Conclusion**

Several conclusions can be drawn from the data available in relation to the research objectives/questions. The first aim of this study was to assess if the pre-service mathematics teachers’ beliefs are consistent with the themes of restraining beliefs from the literature. The data suggest that pre-service mathematics teachers’ beliefs of the learning and teaching of mathematics is quite away from what current philosophies and recommendations of
mathematics instruction about the teaching and learning of mathematics advocate. For example, several students agreed that “If a math problem takes more than 5-10 minutes, it is impossible to solve; every mathematical problem can be solved using a step-by-step procedure or using an equation; math is mostly memorization and so on.” Moreover, many students believe that: “to be good at math, one must be able to solve problems quickly; males are better at math than females; the math teacher should show the exact way to answer the math question one will be tested on”. These observed views agree with several research reports and documents on pre-service mathematics teachers limiting beliefs on mathematics teaching and learning (e.g., Hart, 2002; Wong et al., 2002; Conway and Sloane, 2005). However, Beswick (2007) suggests a well-organized and focused short term continuing professional development as a corrective treatment for such pre-service/in-service teachers’ views.

The second major aim of the research was to test whether hypotheses that the year pre-service mathematics teachers are attending and the set of beliefs pre-service mathematics teachers hold have relationships. Unexpectedly, the data suggests that the set of restraining mathematical beliefs is evenly distributed throughout the years because these beliefs have been established since long, maybe at their early times of learning mathematics. This is supported by many findings and reports (e.g., Mcleod, 1992; Wong et al., 2002; Schoenfeld, 1992; Davis, 1996; Hourigan, And O’Donoghue, 2007). These research findings affirm that once beliefs are established, it is difficult to change them at all. Though more positive positions are demonstrated in the self-efficacy by all the cohorts, year I-III, significant relationship was observed between the years pre-service teachers spend attending and the beliefs they hold.

Moreover, researchers like Davis (1996) and Wong et al., (2002) indicated that the traditional view of the teaching and learning of mathematics has had a negative impact on student beliefs about the subject mathematics and the teaching and learning of mathematics. Mathematics was thought of as a set of rote procedures for symbol manipulation in a certain predetermined way.
The traditional school experience together with the associated teaching practice and the curriculum in use are, at least in part, to blame for these misconceptions. In the end, this type of traditional practice provides students with little opportunity for making sense out of mathematics (Cathcart et al., 2001; Dougherty & Wilson, 1993). As can be understood from research reports globally, such negative beliefs of teachers corrected using different corrective treatments that help to improve negative beliefs toward a more positive and canonical beliefs. For example, Katz (2000) highlights the possible impacts of bad publicity of mathematics and the negative attitude people hold about the subject. He suggests an appropriate intervention such as introducing teachers to the historical development of mathematical ideas to correct such misunderstanding. Similar recommendations were also forwarded by Kenschaft (2005).

**Recommendations**

In the light of the previous discussion and concluding remarks, the following recommendations are suggested.

1. Teacher institutions have to work enthusiastically and meticulously in changing students' views though during the students stay in the institute.
2. Effective enunciation between theory and practice should be a primary concern and has to get common place during course delivery and practices. The research findings reported here can be used as a stepping stone for further investigation to ensure whether teaching methods modeled in classroom challenge the limiting beliefs held by the study in pre-service teachers’ preparation. The constructivist approach to teaching and learning of mathematics cannot be achieved when students practice it within the methods courses only. It should be integrated in all the courses offered to the trainees in the training institute.
3. The teacher institutions must be mindful of timely assessing pre-service teachers’ views about the subjects they are learning and the
subjects that they will teach in schools. In turn, the results of assessment must be used to changing the views of pre-service teachers’ in accordance with the philosophy and recommendations of current classroom instruction.

4. Pre-service mathematics teachers should be involved in classroom research to help them see the difference between the traditional and the more current views on the teaching and learning of mathematics. To do this effectively, pre-service teachers should be changed first.

5. Courses must be designed in ways that provide pre-service mathematics teachers the opportunity to explore the historical developments of major mathematical ideas and developments. This may help to change the pre-service mathematics teachers’ image of mathematics. This approach can be used as a corrective treatment, the potential of which would be the development of positive beliefs and positive attitudes toward mathematics learning and teaching.

6. Attention must be given to the assessment of teacher educators’ beliefs on the nature, teaching and learning of mathematics who are engaged in training pre-service teachers at different levels, to which end they are involved in completing belief questionnaires that measure the themes of beliefs used in this research. If the results of analysis of the beliefs of the teacher educators remain compatible with that of traditional view, continuing professional trainings that support the constructivist classroom instruction must be nurtured to shift their beliefs to a more positive position, because the beliefs teachers espoused can have strong influence on the way they are teaching.
References


